BSJ
BURAPHA SCIENCE JOURNAL
ISSN 2985-0983 (online)
วารสารวิทยาศาสตร์บูรพา ปีที่ 30 (ฉบับที่ 2) พฤษภาคม – สิงหาคม พ.ศ. 2568
BURAPHA SCIENCE JOURNAL Volume 30 (No.2) May – August 2025

บทความวิจัย

Evaluating the Efficiency of Skewness Test Statistics Based on Descriptive Statistics

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Received : 2 April 2025, Received in revised form : 19 June 2025, Accepted : 20 June 2025

Available online : 2 July 2025

Abstract

Background and Objectives: Understanding data distribution is important for identifying outliers. Some values far from most data points may not be real outliers but may naturally occur in certain data types. Traditional boxplots may show many outliers in these cases, even when the values are normal for that data type. Knowing the distribution type and choosing the right boxplot method can help avoid these mistakes. The Ratio Skewed Boxplot method offers a more accurate and reliable approach to outlier detection. This highlights the importance of examining data distribution, particularly by assessing skewness using simple and effective methods. This study examines the efficiency of skewness test statistics derived from descriptive statistics, following Tabor (2010), who proposed eleven test statistics based on the mean, median, first and third quartiles, minimum, maximum, and standard deviation. Although these eleven test statistics were evaluated, no universal critical values can be generally applied to all datasets due to the exact mean and standard deviation of the data are unknown. For this reason, this work aims to determine appropriate critical values and compare the effectiveness of skewness test statistics derived from basic statistical measures that are easy to compute, based on Tabor's research.

Methodology: The study consists of two parts: finding critical values and evaluating efficiency based on Type I error control and power of the test at least 80%. Critical values for eleven test statistics were determined using a Monte Carlo simulation with 100,000 iterations. Data following a normal distribution were generated with sample sizes of n = 10, 20, 30, 50, 100, and 200, and the values for all test statistics were calculated. The 95th and 99th percentiles of the sampling distributions for all test statistics were identified as the critical values corresponding to significance levels of 0.05 and 0.01, respectively. This process was repeated using different parameter sets for the normal distribution, and the results were averaged for each sample size. In order to calculate the Type I error, we generated data from a standard normal distribution with 10,000 simulation replicates. All tests were performed based on the critical values provided in the first part. The Type I error rate was calculated as the proportion of rejections, determined by dividing the number of rejections by the total number of replications. Another key criterion is the power of the test. Data were simulated from three skewed distributions: the chi-square distribution, the gamma

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distribution, and the lognormal distribution using multiple parameter settings to represent various characteristics of the data. The test power is determined by the probability of rejecting the null hypothesis. A test statistic with the highest power is considered the most efficient.

Main Results: The critical values of the test statistics obtained from certain experimental methods are applicable due to their derivation through data simulation using nine parameter settings. It is observed that if the test statistics derived from all parameter sets are similar, the averaged critical values will be appropriate and enable the test statistics to effectively control Type I error. The results indicate that Type I error rates for most skewness test statistics fall within Bradley's liberal Type I error range. This range is (0.025, 0.075) for a 0.05 significance level and (0.005, 0.015) for a 0.01 significance level across all sample sizes. This means that they are close to the significance level. In other words, most test statistics have been proven to be efficient methods. Next, we considered only test statistics that effectively control Type I error and compared their power of the test. It can be seen that test statistics B, D, and I, as shown in Table 1, exhibit the highest power and achieve at least 80% power of the test when the sample size is large. Additionally, their efficiency depends on the skewness coefficient of the data. The results exhibit a consistent pattern across all studied data distributions, including the chi-square distribution, the gamma distribution, and the lognormal distribution.

Conclusions: The power of the test for the three best methods depends on the shape of the distribution and the sample size. A highly skewed distribution gives strong power of the test, over 80%, even with a small sample. In contrast, less skewed distributions require larger samples to achieve a test power of 0.80. This study suggests that the test statistics can be effective when the sample size is sufficiently large, ensuring the test meets the 80% threshold and accurately detects skewness. Additionally, the three test statistics were applied to real data, specifically wind speed data from two locations in Thailand: the Nakhon Ratchasima Meteorological Station and the Chaiyaphum Meteorological Station. The study analyzed 100 wind speed data points collected between March 1, 2023 and June 8, 2023. The data were used to create histograms to examine the distribution characteristics. Then, the three skewness test statistics were applied to the data. The results were consistent across all three methods, indicating that the wind speed data from both stations exhibit a skewed distribution.

Keywords: critical values ; descriptive statistics ; power of the test ; skewness test ; Type I error

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Introduction

Understanding data distribution is crucial, especially for identifying outliers. While outliers can introduce errors in analysis, some values far from the majority of data points may not be true outliers but instead occur naturally in certain types of data. Skewed distributions, particularly right-skewed ones, are common in important datasets. For instance, wind speed data in Turkey follows gamma and lognormal distributions (Özkan *et al*., 2020). Similarly, wind speed data in Rwanda also exhibits positive skewness. In most areas, wind speeds are generally low to moderate during the day, with strong gusts occurring only occasionally. Although rare, this pattern is natural and often results in a distribution that follows the gamma, lognormal, or Weibull distribution (Safari, 2011).Using traditional boxplots for these data types may show many outliers. In reality, these values are normal for that data type. Therefore, knowing the distribution type in advance and selecting an appropriate boxplot method can avoid mistakes in identifying outliers. The Ratio Skewed Boxplot method (Walker *et al*., 2018) is a suitable approach, providing more accurate and reliable outlier detection. Therefore, it is clear that examining the data distribution is very important. In particular, testing data skewness using simple and straightforward methods is essential. Tabor (2010) introduced eleven skewness test statistics based on descriptive statistics, including the minimum, first quartile, median, third quartile, and maximum. These descriptive statistics are used to calculate skewness test statistics, as shown in Table 1. Tabor's study evaluates the performance of these skewness test statistics under a normal distribution with a mean of 5 and a standard deviation of 1. All skewness test methods are applied, using the 95th percentile as the critical value. Additionally, the study examines the performance of these statistics under a Chi-square distribution that was rescaled to have the same mean and standard deviation. However, Tabor (2010) did not provide clear critical values for testing data from distributions with varying means and standard deviations. In practice, the exact population parameters of the sampled data are unknown. Therefore, this study aims to determine the critical values for the eleven skewness test statistics and evaluate their performance. The performance of the test is based on Type I error control and power of the test. The findings will enhance the applicability of skewness tests to real-world data analysis. Moreover, this study serves as a guideline for the effective use of the skewness coefficient, providing an additional option for preliminary data examination to determine whether the distribution is skewed.

Methodology

This research is divided into two parts: calculating the critical value and testing the efficiency of the test statistic. Critical values for eleven test statistics were determined using a Monte Carlo simulation with 100,000 iterations. Data following a normal distribution were generated for sample sizes of n = 10, 20, 30, 50, 100, and 200, and the values for all test statistics were calculated. The 95th and 99th percentiles of the sampling distributions for all test statistics were identified as the critical values corresponding to significance levels of 0.05 and 0.01, respectively. This process was repeated using different parameter sets for the normal distribution $N\left(\mu,\sigma^2\right)$ that are N(2,1), N(5,1), N(10,1), N(2,4), N(5,4), N(10,4), N(2,16), N(5,16), and N(10,16). In order to cover a wide range of data characteristics with respect to both central tendency and variability.

Table 1 Eleven test statistics (Tabor, 2010)

Name	Statistic	Name	Statistic	Name	Statistic
A	$\frac{mean}{median}$	B	$\frac{\max - median}{median - \min}$	C	$\frac{Q_3 - median}{median - Q_1}$
D	$\frac{\max - Q_3}{Q_1 - \min}$	E	$\frac{\frac{1}{2}(\min + \max)}{median}$	F	$\frac{\frac{1}{2}(Q_1 + Q_3)}{median}$
G	$\frac{\frac{1}{2}(\min + \max)}{\frac{1}{2}(Q_1 + Q_3)}$	H	$\frac{\min + Q_1 + median + Q_3 + \max}{5}$	I	$\frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3}{\left(\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}\right)^3}$
J	$\frac{3(mean - median)}{SD}$	K	$\frac{(Q_3 - median) - (median - Q_1)}{Q_3 - Q_1}$		

To evaluate the efficiency of the test statistics, the criterion used is its ability to control Type I error. We examine Type I error rates by generating Normal distributions from 10,000 simulation replicates using parameters N(0,1). All tests are performed based on the critical values provided in Tables 2-3. The Type I error rate is calculated as the proportion of rejections, determined by dividing the number of rejections by the total number of replications. Another criterion is the power of the test, which will only be considered for test statistics that adequately control Type I error. In other words, the type I error rate should fall within Bradley's liberal Type I error bounds (Bradley, 1978). To determine the power of the test, data will be simulated with a skewed distribution. The null hypothesis assumes a normal distribution, while the alternative hypothesis assumes a skewed distribution.

1) *Chi-square distribution*

Chi-square distribution has the probability density function as

$$f(x) = \frac{x^{\frac{k}{2}-1} e^{-\frac{x}{2}}}{2^{\frac{k}{2}}\Gamma\left(\frac{k}{2}\right)} \quad ; \quad x \ge 0 \tag{1}$$

The mean is k and the variance is $2k$, k is called degrees of freedom.

2) *Gamma distribution*

Gamma distribution has the probability density function as

$$f(x)=\frac{x^{\alpha-1}e^{-\frac{x}{\beta}}}{\Gamma(\alpha)\beta^{\alpha}} \quad ; \quad x\geq 0,\alpha>0,\beta>0 \tag{2}$$

The mean is $\alpha\beta$ and the variance is $\alpha\beta^2$.

3) *Lognormal distribution*

Lognormal distribution has the probability density function as

$$f(x)=\frac{1}{x\sigma\sqrt{2\pi}}\exp\left(-\frac{(\ln x-\mu)^2}{2\sigma^2}\right) \quad ; \quad x\geq 0,\mu>0,\sigma>0 \tag{3}$$

The mean is $\exp\left(\mu+\frac{\sigma^2}{2}\right)$ and the variance is $\left[\exp\left(\sigma^2\right)-1\right]\left[\exp\left(2\mu+\sigma^2\right)\right]$.

The Type I error rate is calculated using the following algorithm:

- Generate samples of size n from the standard normal distribution.
- Calculate all test statistics and compare them with critical values. If the test statistic is greater than the critical value, then reject the null hypothesis.
- Repeat for 10,000 iterations.
- Compute the proportion of rejections of H_0.

The simulation for the power of the test follows the algorithm below:

- Specify the parameters of each distribution (Chi-squared, Gamma, Lognormal distribution).
- Generate samples of size n from the specified distribution.
- Calculate all test statistics and compare them with critical values. If the test statistic is greater than the critical value, then reject the null hypothesis.
- Repeat for 10,000 iterations.
- Compute the proportion of rejections of H_0.

Results

The results are divided into two parts. The first part determines the critical value at significance levels of 0.05 and 0.01. The second part examines the efficiency of the test statistics using Type I error control and power of the test as criteria.

1. Critical values

The critical values obtained across different parameter sets were then averaged for each sample size and are shown in Tables 2 and 3

2. Type I error rates

The Type I error rate is measured as the ratio of the number of rejections to the total replications. This rate is illustrated in Table 4 and 5. The results show that the Type I error rates for B, C, D, I, J, and K consistently fall within Bradley's liberal Type I error range. This range is (0.025, 0.075) for a 0.05 significance level and (0.005, 0.015) for a 0.01 significance level across all sample sizes. This means that they are closer to the significance level. In other words, test statistics B, C, D, I, J and K are proven to be efficient methods.

3. The power of the test

In this section, we generated three types of distributions: Chi-squared, Lognormal, and Gamma distributions, to represent skewed data. Additionally, we used different parameter sets of each distribution. By setting parameters according to different skewness levels, some parameter values are referenced from wind speed research mentioned in the introduction. These parameters are presented in Table 6, and the estimated power of the test is shown in Figures 1-3.

Table 2 Critical values at significance level of 0.05

Test	Sample size					
	10	20	30	50	100	200
A	1.3588	1.2616	1.2050	1.1444	1.0919	1.0611
B	2.7284	2.0592	1.8414	1.6598	1.5033	1.4061
C	3.7679	2.6513	2.2464	1.8814	1.5615	1.3721
D	4.4151	2.7913	2.3356	1.9935	1.7204	1.5593
E	1.7769	1.6348	1.5411	1.4429	1.3589	1.3144
F	1.3803	1.2902	1.2311	1.1663	1.1076	1.0728
G	1.6138	1.5105	1.4528	1.3928	1.3364	1.3036
H	6.9103	6.5982	6.4658	6.3436	6.2287	6.1547
I	0.9513	0.7729	0.6619	0.5344	0.3892	0.2798
J	0.9679	0.7574	0.6392	0.5081	0.3650	0.2613
K	0.5805	0.4522	0.3839	0.3059	0.2192	0.1569

In this study, test statistics are considered effective only if their power is at least 80% (Cohen, 1988). The C, J, and K test statistics do not achieve the 80% test power criterion. Additionally, C and K have very similar power of the test. However, none of the three methods are effective enough for testing skewness. The results are as follows:

Table 3 Critical values at significance level of 0.01

Test	Sample size					
	10	20	30	50	100	200
A	2.2076	1.7764	1.5343	1.2904	1.1495	1.0931
B	4.2975	2.8069	2.3833	2.0489	1.7824	1.6245
C	7.2398	4.0778	3.2045	2.4601	1.8820	1.5646
D	8.8666	4.4510	3.3949	2.6667	2.1536	1.8760
E	3.6063	2.7990	2.2883	1.8246	1.5702	1.4771
F	2.2856	1.8609	1.5749	1.3232	1.1736	1.1100
G	2.9830	2.2915	1.9145	1.6618	1.5204	1.4563
H	7.4257	6.9859	6.8075	6.6312	6.4768	6.3718
I	1.4046	1.1554	0.9884	0.7879	0.5665	0.4028
J	1.3051	1.0475	0.8889	0.7117	0.5137	0.3681

3.1 Chi-square distribution

The B, D and I test statistics show similar testing power. These test statistics become effective when the sample size is large ($n \geq 100$). Additionally, their effectiveness depends on the level of skewness. The test statistics maintain good efficiency with larger sample sizes as the skewness coefficient decreases. However, for data following a Chi-square distribution with 5 degrees of freedom, these test statistics provide similarly good efficiency, requiring a sample size of at least 50

3.2 Gamma distribution

In the case of the Gamma distribution with parameters (2.99, 1.33) and (3.44, 0.85), the B, D, and I test statistics exhibit similar testing power and meet Cohen's criteria when the sample size is at least 50. However, a sample size of at least 100 is required to maintain the efficiency of the B, D, and I test statistics when using a significance level of 0.01. For the parameters (13.29, 4.64), only the I test statistic demonstrates the highest efficiency at a 0.05 significance level, requiring a sample size of at least 200.

3.3 Lognormal distribution

The B, D and I test statistics provide similar testing power and achieve at least 80% power when the sample size is no less than 50 for parameters (0.66, 0.54) and (1.27, 0.5). For the parameter (1.02, 0.26), the B, D and I test statistics maintain good efficiency when using a sample size of at least 200. However, the B and I test statistic shows good efficiency with a sample size of 100 when tested at a significance level of 0.05.

Discussion

The results show that the power of the test depends on the shape of the distribution and the sample size. A highly skewed distribution gives strong power of the test, over 80%, even with a small sample size. In contrast, less skewed distributions require larger samples to achieve a test power of 0.80. In other words, the test performs better with a highly skewed distribution. This study suggests that the test statistics can be effective when the sample size is sufficiently large, ensuring the test meets the 80% threshold and accurately detects skewness. However, these test statistics are easily computed and can be applied for preliminary outlier detection. Additionally, these findings provide a guideline for selecting an appropriate box plot for skewed data, improving the accuracy of outlier identification.

Table 4 Type I error rates for the significance level 0.05

Test	Sample size					
	10	20	30	50	100	200
A	0.1927	0.2171	0.2290	0.2456	0.2620	0.1483
B	**0.0501**	**0.0485**	**0.0493**	**0.0494**	**0.0489**	**0.0513**
C	**0.0492**	**0.0502**	**0.0495**	**0.0493**	**0.0498**	**0.0492**
D	**0.0504**	**0.0498**	**0.0504**	**0.0493**	**0.0491**	**0.0510**
E	0.2130	0.2599	0.2867	0.3226	0.3628	0.3941
F	0.2117	0.2400	0.2578	0.2749	0.2934	0.3032
G	0.2642	0.3149	0.3394	0.3657	0.3982	0.4283
H	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
I	**0.0506**	**0.0500**	**0.0494**	**0.0498**	**0.0482**	**0.0530**
J	**0.0491**	**0.0493**	**0.0495**	**0.0497**	**0.0506**	**0.0510**
K	**0.0492**	**0.0503**	**0.0495**	**0.0493**	**0.0498**	**0.0492**

Note : Bold and underlined letters indicate being within the Bradley's criteria

Table 5 Type I error rates for the significance level 0.01

Test	Sample size					
	10	20	30	50	100	200
A	0.0915	0.1307	0.1610	0.1995	0.2410	0.2625
B	**0.0105**	**0.0100**	**0.0104**	**0.0106**	**0.0079**	**0.0080**
C	**0.0105**	**0.0097**	**0.0102**	**0.0090**	**0.0102**	**0.0090**
D	**0.0092**	**0.0110**	**0.0091**	**0.0104**	**0.0089**	**0.0093**
E	0.1055	0.1648	0.2177	0.2789	0.3395	0.3770
F	0.0936	0.1513	0.1843	0.2250	0.2717	0.2963
G	0.1355	0.2216	0.2873	0.3399	0.3771	0.4045
H	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
I	**0.0092**	**0.0097**	**0.0109**	**0.0094**	**0.0097**	**0.0088**
J	**0.0116**	**0.0095**	**0.0088**	**0.0113**	**0.0098**	**0.0110**
K	**0.0105**	**0.0102**	**0.0102**	**0.0090**	**0.0102**	**0.0089**

Note : Bold and underlined letters indicate being within the Bradley's criteria

Table 6 Parameter of skewed distributions

Distribution	Parameters
Chi-squared (k)	5, 8, 10
Gamma (α,β)	(2.99, 1.33), (3.44, 0.85), (13.29, 4.64)
Lognormal (μ,σ)	(0.66, 0.54), (1.27, 0.5), (1.02, 0.26)

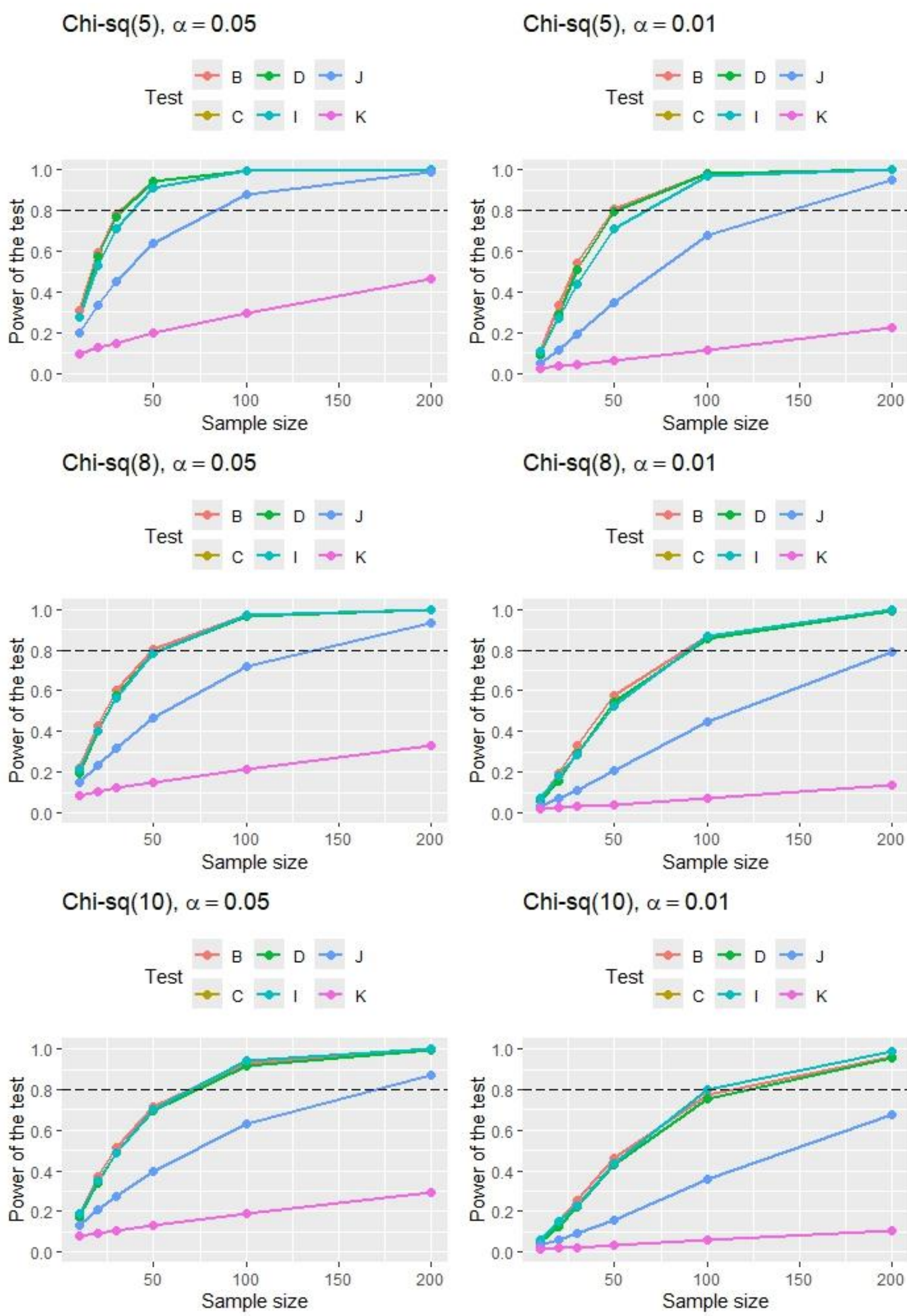

Figure 1 The estimated power of the test for Chi-square distribution data

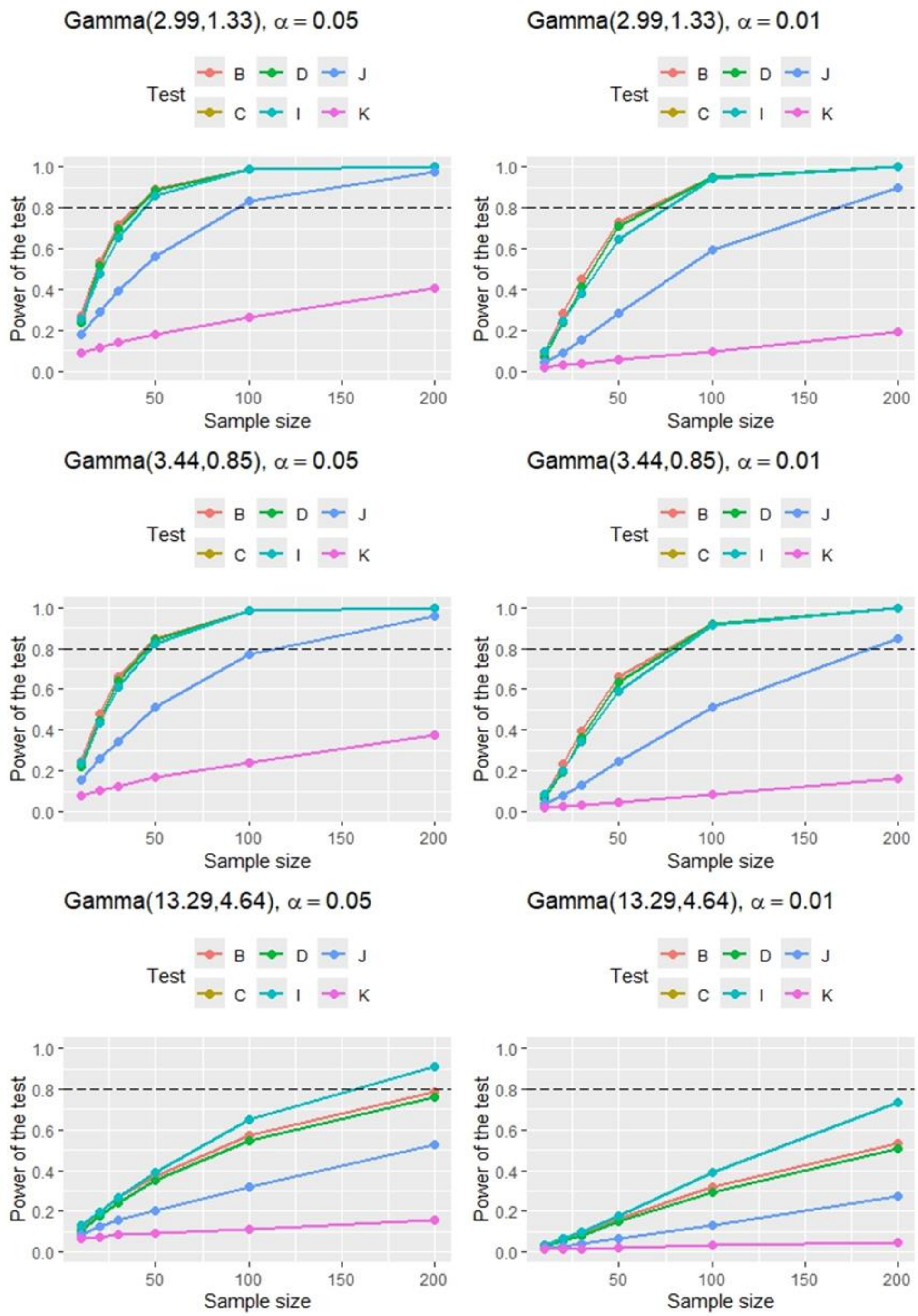

Figure 2 The estimated power of the test for Gamma distribution data

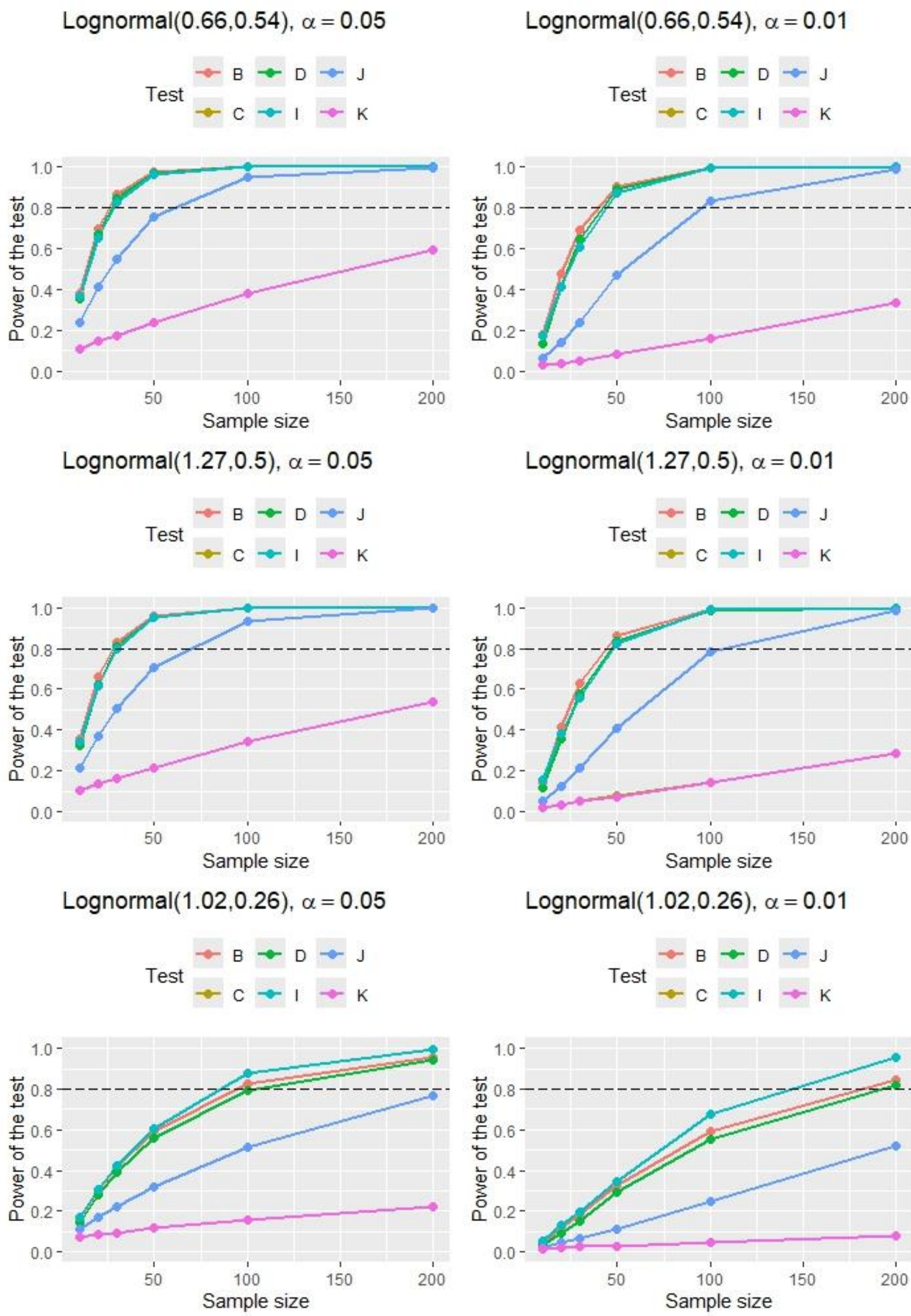

Figure 3 The estimated power of the test for Lognormal distribution data

Based on Tabor's study, the five methods with the highest statistical power were ranked as B, E, I, G, and D, respectively. However, that study did not consider the criterion of Type I error control. When both criteria—Type I error rate control and statistical power—are considered, it becomes clear that methods E and G fail to maintain the nominal significance level, making them unsuitable for reliable hypothesis testing. Therefore, the findings of the present study, which show that methods B, D, and I have both high statistical power and acceptable Type I error rate control, suggest that these methods are the most efficient. This conclusion is consistent with Tabor's research.

In the application of statistical tests for skewness to real-world data, as mentioned in the introduction, the wind speed data have a right-skewed distribution. Therefore, we used wind speed data from the Thai Meteorological Department at the Nakhon Ratchasima and Chaiyaphum meteorological stations. These places are important locations for wind turbine installation in Thailand to produce clean energy. The study analyzed 100 wind speed data points collected between March 1, 2023 and June 8, 2023. This period covers the summer to early rainy season in Thailand. A histogram of the data was created, as shown in Figures 4-5. Descriptive statistics and the B, D, and I test statistics were calculated and presented in Tables 7-8.

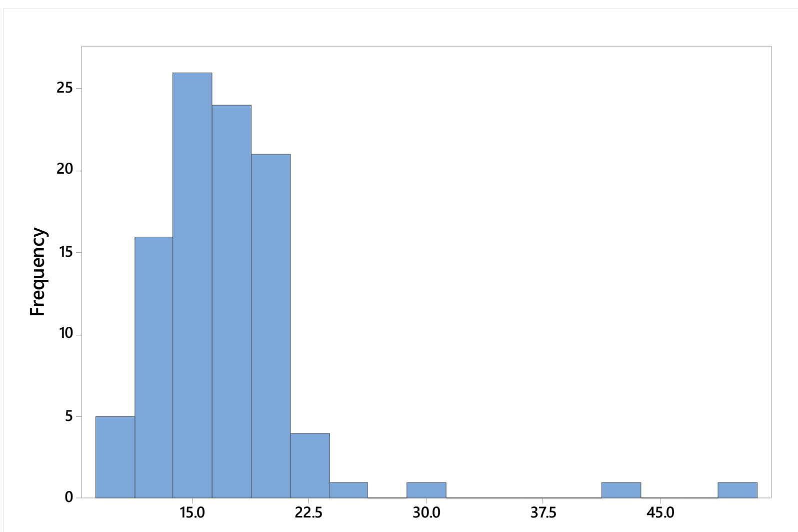

Figure 4 Histogram of wind speed from Nakhon Ratchasima meteorological station

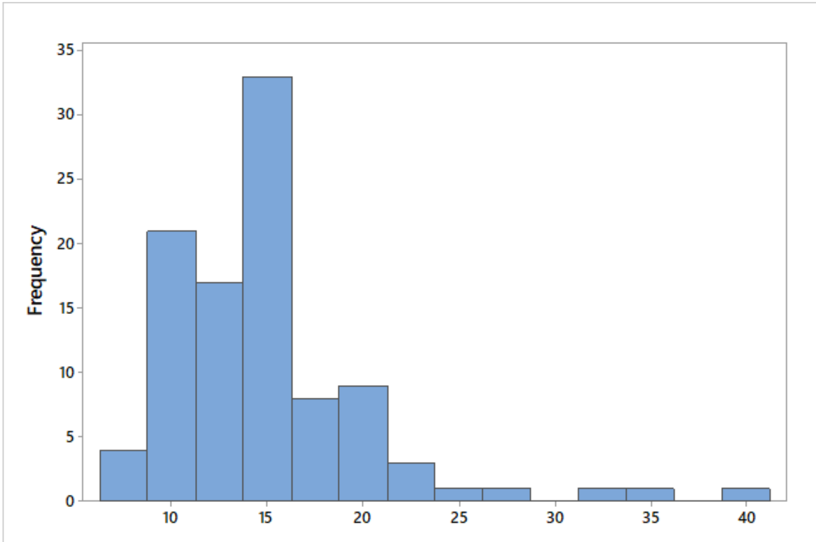

Figure 5 Histogram of wind speed from Chaiyaphum meteorological station

Table 7 Descriptive Statistics of wind speed

Descriptive Statistics	Nakhon Ratchasima	Chaiyaphum
Mean	17.09	14.94
Standard deviation	5.507	5.481
Median	17	15
Minimum	10	8
Maximum	51	40
First Quartile	14	11.25
Third Quartile	19	16.75
Skewness	3.48	2.02

Table 8 Skewness test statistics

Station	Skewness test		
	B	D	I
Nakhon Ratchasima	4.857**	8.000**	3.428**
Chaiyaphum	3.571**	7.154**	1.992**

Note : ** significant at the 0.01 level

The results show that all test statistics lead to the same conclusion. The wind speed data from both locations have a skewed distribution. This is consistent with the histogram and the skewness coefficient of the data.

Conclusions

The eleven skewness test statistics for data following Chi-square, Gamma, and Lognormal distributions with any mean and variance can be applied using the critical values provided in Tables 2-3. However, only the test statistics B, C, D, I, J, and K effectively control Type I error, whereas test statistics A, E, F, G, and H fail to meet this criterion. Therefore, the power of the test analysis considers only the test statistics that can control Type I error. The results show that test statistics B, D, and I exhibit the highest and most similar test power when the sample size is large. Additionally, test statistics C and K demonstrate the same and lowest power of the test in all cases, as shown in Figures 1-3.

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